Generalized uncertainty principle, quantum gravity and Hořava-Lifshitz gravity

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Abstract

We investigate a close connection between generalized uncertainty principle (GUP) and deformed Hořava-Lifshitz (HL) gravity. The GUP commutation relations correspond to the UV-quantum theory, while the canonical commutation relations represent the IR-quantum theory. Inspired by this UV/IR quantum mechanics, we obtain the GUP-corrected graviton propagator by introducing UV-momentum $p_i = p_{0i}(1 + \beta p_0^2)$ and compare this with tensor propagators in the HL gravity. Two are the same up to p_0^4-order.

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1 Introduction

Recently Hořava has proposed a renormalizable theory of gravity at a Lifshitz point [1], which may be regarded as a UV complete candidate for general relativity. At short distances the theory of $z = 3$ Hořava-Lifshitz (HL) gravity describes interacting nonrelativistic gravitons and is supposed to be power counting renormalizable in (1+3) dimensions. Recently, the HL gravity theory has been intensively investigated in [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. The equations of motion were derived for $z = 3$ HL gravity [29, 30], and its black hole solution was first found in asymptotically anti-de Sitter spacetimes [30] and black hole in asymptotically flat spacetimes [31].

It seems that the GUP-corrected Schwarzschild black hole is closely related to black holes in the deformed Hořava-Lifshitz gravity [32, 33]. Also, the GUP provides naturally a UV cutoff to the local quantum field theory as quantum gravity effects [34, 35].

On the other hand, one of main ingredients for studying quantum gravity is the GUP, which has been argued from various approaches to quantum gravity and black hole physics [36]. Certain effects of quantum gravity are universal and thus, influence almost any system with a well-defined Hamiltonian [37]. The GUP satisfies the modified Heisenberg algebra [38]

$$[x_i, p_j] = i\hbar\Big(\delta_{ij} + \beta p^2\delta_{ij} + 2\beta p_i p_j\Big),\ [x_i, x_j] = [p_i, p_j] = 0 \tag{1}$$

where p_i is considered as the momentum at high energies and thus, it can be interpreted to be the UV-commutation relations. Here $p^2 = p_i p_i$. In this case, the minimal length which follows from these relations is given by

$$\delta x_{\rm min} = \hbar\sqrt{5\beta}. \tag{2}$$

On the other hand, introducing IR-canonical variable p_{0i} with $x_i = x_{0i}$ through the replacement

$$p_i = p_{0i}\Big(1 + \beta p_0^2\Big), \tag{3}$$

these variables satisfy canonical commutation relations

$$[x_{0i}, p_{0j}] = i\hbar\delta_{ij},\ [x_{0i}, x_{0j}] = [p_{0i}, p_{0j}] = 0. \tag{4}$$

Here p_{0i} is considered as the momentum at low energies with $p_0^2 = p_{0i}p_{0i}$. It is easy to show that Eq. (1) is satisfied to linear-order β when using Eq. (4). Hence, the replacement (3) could be used as an important low-energy window to investigate quantum gravity phenomenology up to linear-order β.

It was known for deformed HL gravity that the UV-propagator for tensor modes t_{ij} take a complicated form Eq. (32), including up to p_0^6-term from the Cotton bilinear term $C_{ij}C_{ij}$. We have explored a connection between the GUP commutator and the deformed HL gravity [39]. Explicitly, we have replaced a relativistic cutoff function $\mathcal{K}(\frac{p^2}{\Lambda^2})$ by a non-relativistic density function $\mathcal{D}_D(\beta\vec{p}^2)$ to derive GUP-corrected graviton propagators. These were compared to (32). It was pointed out that two are *qualitatively similar*, but the p^5-term arisen from the crossed term of Cotton and Ricci tensors did not appear in the GUP-corrected propagators. Also, it was unclear why the $D = 2$ GUP-corrected tensor propagator (not the $D = 3$ GUP-corrected propagator) is similar to the UV-propagator derived from the $z = 3$ HL gravity.

In this work, we investigate a close connection between GUP and deformed HL gravity. At high energies, we assume that the UV-propagator takes the conventional form $G_{\rm UV}(\varpi, p^2)$ in Eq. (34), whereas at low energies, the IR-propagator takes the conventional form $G_{\rm IR}(\varpi, p_0^2)$ in Eq. (35). It is very important to understand how the UV-propagator is related to the IR-propagator in the non-relativistic gravity theory. We find a GUP-corrected graviton propagator by applying (3) to $G_{\rm UV}(\varpi, p^2)$ and compare it with the UV-tensor propagator (32) in the HL gravity. Two are *the same* up to p_0^4-order, although the p_0^5-term arisen from a crossed term of Cotton tensor and Ricci tensor is still missed in the GUP-corrected graviton propagator. This indicates that a power-counting renormalizable theory of the HL gravity is closely related to the GUP.

2 $z = 3$ HL gravity

Introducing the ADM formalism where the metric is parameterized

$$ds^2_{ADM} = -N^2dt^2 + g_{ij}\Big(dx^i - N^idt\Big)\Big(dx^j - N^jdt\Big)\,, \tag{5}$$

the Einstein-Hilbert action can be expressed as

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{g}N\Big[K_{ij}K^{ij} - K^2 + R - 2\Lambda\Big], \tag{6}$$

where G is Newton's constant and extrinsic curvature K_{ij} takes the form

$$K_{ij} = \frac{1}{2N}\Big(\dot{g}_{ij} - \nabla_iN_j - \nabla_jN_i\Big)\,. \tag{7}$$

Here, a dot denotes a derivative with respect to t. An action of the non-relativistic renormalizable gravitational theory is given by [1]

$$S_{HL} = \int dtd^3x\Big[\mathcal{L}_K + \mathcal{L}_V\Big], \tag{8}$$

where the kinetic terms are given by

$$\mathcal{L}_K = \frac{2}{\kappa^2}\sqrt{g}NK_{ij}\mathcal{G}^{ijkl}K_{kl} = \frac{2}{\kappa^2}\sqrt{g}N\Big(K_{ij}K^{ij} - \lambda K^2\Big), \tag{9}$$

with the DeWitt metric

$$\mathcal{G}^{ijkl} = \frac{1}{2}\Big(g^{ik}g^{jl} - g^{il}g^{jk}\Big) - \lambda g^{ij}g^{kl} \tag{10}$$

and its inverse metric

$$\mathcal{G}_{ijkl} = \frac{1}{2}\Big(g_{ik}g_{jl} - g_{il}g_{jk}\Big) - \frac{\lambda}{3\lambda - 1}g_{ij}g_{kl}. \tag{11}$$

The potential terms is determined by the detailed balance condition as

$$\begin{aligned}\mathcal{L}_V = -\frac{\kappa^2}{2}\sqrt{g}NE^{ij}\mathcal{G}_{ijkl}E^{kl} &= \sqrt{g}N\Bigg\{\frac{\kappa^2\mu^2}{8(1-3\lambda)}\Big(\frac{1-4\lambda}{4}R^2 + \Lambda_W R - 3\Lambda_W^2\Big) \\ &\quad - \frac{\kappa^2}{2\eta^4}\left(C_{ij} - \frac{\mu\eta^2}{2}R_{ij}\right)\left(C^{ij} - \frac{\mu\eta^2}{2}R^{ij}\right)\Bigg\}.\end{aligned} \tag{12}$$

Here the E tensor is defined by

$$E^{ij} = \frac{1}{\eta^2}C^{ij} - \frac{\mu}{2}\Big(R^{ij} - \frac{R}{2}g^{ij} + \Lambda_W g^{ij}\Big) \tag{13}$$

with the Cotton tensor C_{ij}

$$C^{ij} = \frac{\epsilon^{ik\ell}}{\sqrt{g}}\nabla_k\left(R^j{}_\ell - \frac{1}{4}R\delta^j_\ell\right). \tag{14}$$

Explicitly, E_{ij} could be derived from the Euclidean topologically massive gravity

$$E^{ij} = \frac{1}{\sqrt{g}}\frac{\delta W_{TMG}}{\delta g_{ij}} \tag{15}$$

with

$$W_{TMG} = \frac{1}{\eta^2}\int d^3x\epsilon^{ikl}\Big(\Gamma^m_{il}\partial_j\Gamma^l_{km} + \frac{2}{3}\Gamma^n_{il}\Gamma^l_{jm}\Gamma^m_{kn}\Big) - \mu\int d^3x\sqrt{g}(R - 2\Lambda_W), \tag{16}$$

where ϵ^{ikl} is a tensor density with $\epsilon^{123} = 1$.

In the IR limit, comparing $\mathcal{L}_0$ with Eq.(6) of general relativity, the speed of light, Newton's constant and the cosmological constant are given by

$$c = \frac{\kappa^2\mu}{4}\sqrt{\frac{\Lambda_W}{1-3\lambda}}, \qquad G = \frac{\kappa^2}{32\pi\, c}, \qquad \Lambda_{cc} = \tfrac{3}{2}\Lambda_W\,. \tag{17}$$

The equations of motion were derived in [29] and [30]. We would like to mention that the IR vacuum of this theory is anti-de Sitter (AdS_4) spacetimes. Hence, it is interesting to

take a limit of the theory, which may lead to a Minkowski vacuum in the IR sector. To this end, one may deform the theory by introducing "$\mu^4 R$" ($\tilde{\mathcal{L}}_V = \mathcal{L}_V + \sqrt{g}N\mu^4 R$) and then, take the $\Lambda_W \to 0$ limit [31]. We call this the deformed HL gravity without detailed balance condition. This does not alter the UV properties of the theory, while it changes the IR properties. That is, there exists a Minkowski vacuum, instead of an AdS vacuum. In the IR limit, the speed of light and Newton's constant are given by

$$c^2 = \frac{\kappa^2\mu^4}{2},\ G = \frac{\kappa^2}{32\pi\, c},\ \lambda = 1. \tag{18}$$

The deformed HL gravity has an important parameter [31]

$$\omega = \frac{8\mu^2(3\lambda - 1)}{\kappa^2}, \tag{19}$$

which takes the form for $\lambda = 1$

$$\omega = \frac{16\mu^2}{\kappa^2}. \tag{20}$$

Actually, $\frac{1}{2\omega}$ plays the role of a charge in the Kehagias-Sfetsos (KS) black hole with $\lambda = 1$ and $K_{ij} = C_{ij} = 0$ [32] derived from the Lagrangian

$$\tilde{\mathcal{L}}_V^{\lambda=1} = \sqrt{g}N\mu^4\Big(R + \frac{3}{4\omega}R^2 - \frac{2}{\omega}R_{ij}R_{ij}\Big). \tag{21}$$

and a spherically symmetric metric ansatz. Furthermore, it was shown that the entropy of KS black hole could be explained from the entropy of GUP-corrected Schwarzschild black hole when making a connection of $\beta \to \frac{1}{\omega}$ [33].

3 GUP-quantum mechanics

A meaningful prediction of various theories of quantum gravity (string theory) and black holes is the presence of a minimum measurable length or a maximum observable momentum. This has provided the generalized uncertainty principle which modifies commutation relations shown by Eq. (1). A universal quantum gravity correction to the Hamiltonian is given by

$$\mathcal{H}_{UV} = \frac{p^2}{2m} + V(x_i) = \frac{p_0^2}{2m} + V(x_{0i}) + \frac{\beta}{m}p_0^4 + \frac{\beta^2}{2m}p_0^6 \tag{22}$$

$$\equiv \mathcal{H}_{IR} + \mathcal{H}_1 \tag{23}$$

with

$$\mathcal{H}_{IR} = \frac{p_0^2}{2m} + V(x_{0i}), \quad \mathcal{H}_1 = \frac{\beta}{m}p_0^4 + \frac{\beta^2}{2m}p_0^6. \tag{24}$$

We note that Eq. (23) may be used for a perturbation study with $p_0 = -i\hbar d/dx_{0i}$. We see that any system with a well-defined quantum (or even classical) Hamiltonian $\mathcal{H}_{IR}$, is perturbed by $\mathcal{H}_1$ near the Planck scale. In this sense, the quantum gravity effects are in some sense universal. Some examples were performed in [37, 40, 41, 42]. It turned out that the corrections could be interpreted in two ways when considering linear-order perturbation $\mathcal{H}_1 = \frac{\beta}{m}p_0^4$: either that for $\beta = \beta_0 l_{\rm Pl}^2/2\hbar^2$ with $\beta_0 \sim 1$, they are exceedingly small, beyond the reach of current experiments or that they predict upper bounds on the quantum gravity parameter $\beta_0 \leq 10^{34}$ for the Lamb shift.

3.1 Tensor modes for deformed $z = 3$ HL gravity

The field equation for tensor modes propagating on the Minkowski spacetimes is given by [24]

$$\ddot{t}_{ij} - \frac{\mu^4\kappa^2}{2} \triangle t_{ij} + \frac{\mu^2\kappa^4}{16} \triangle^2 t_{ij} - \frac{\mu\kappa^4}{4\eta^2}\epsilon_{ilm}\partial^l \triangle^2 t_j{}^m - \frac{\kappa^4}{4\eta^4} \triangle^3 t_{ij} = T_{ij} \tag{25}$$

with external source T_{ij} and the Laplacian $\triangle = \partial_i^2 \to -p_0^2$. We could not obtain the covariant propagator because of the presence of ϵ-term. Assuming a massless graviton propagation along the x^3-direction with $p_{0i} = (0, 0, p_3)$, then the t_{ij} can be expressed in terms of polarization components as [28]

$$t_{ij} = \begin{pmatrix} t_+ & t_\times & 0 \\ t_\times & -t_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{26}$$

Using this parametrization, we find two coupled equations for different polarizations

$$\ddot{t}_+ - \frac{\mu^4\kappa^2}{2} \triangle t_+ + \frac{\kappa^4\mu^2}{16} \triangle^2 t_+ + \frac{\kappa^4\mu}{4\eta^2}\partial_3 \triangle^2 t_\times - \frac{\kappa^4}{4\eta^4} \triangle^3 t_+ = T_+, \tag{27}$$

$$\ddot{t}_\times - \frac{\mu^4\kappa^2}{2} \triangle t_\times + \frac{\kappa^4\mu^2}{16} \triangle^2 t_\times - \frac{\kappa^4\mu}{4\eta^2}\partial_3 \triangle^2 t_+ - \frac{\kappa^4}{4\eta^4} \triangle^3 t_\times = T_\times. \tag{28}$$

In order to find two independent components, we introduce the left-right base defined by

$$t_{L/R} = \frac{1}{\sqrt{2}}\Big(t_+ \pm i t_\times\Big) \tag{29}$$

where $t_L(t_R)$ represent the left (right)-handed modes. After Fourier-transformation, we find two decoupled equations

$$-\varpi^2 t_L + c^2 p_0^2 t_L + \frac{\kappa^4\mu^2}{16}(p_0^2)^2 t_L - \frac{\kappa^4\mu}{4\eta^2}p_3(p_0^2)^2 t_L + \frac{\kappa^4}{4\eta^4}(p_0^2)^3 t_L = T_L, \tag{30}$$

$$-\varpi^2 t_R + c^2 p_0^2 t_R + \frac{\kappa^4\mu^2}{16}(p_0^2)^2 t_R + \frac{\kappa^4\mu}{4\eta^2}p_3(p_0^2)^2 t_R + \frac{\kappa^4}{4\eta^4}(p_0^2)^3 t_R = T_R. \tag{31}$$

We have UV-tensor propagators with $\omega = 16\mu^2/\kappa^2$

$$t_{L/R} = -\frac{T_{L/R}}{\varpi^2 - c^2\Big(p_0^2 + \frac{2}{\omega}p_0^4 \mp \frac{8}{\eta^2\mu\omega}p_3p_0^4 + \frac{128}{\eta^4\kappa^2\omega^2}p_0^6\Big)}. \tag{32}$$

We note that the left-handed mode is not allowed because it may give rise to ghost $(-\frac{8c^2}{\eta^2\mu\omega}p_3p_0^4)$, while the right-handed mode is allowed because there is no ghost $(\frac{8c^2}{\eta^2\mu\omega}p_3p_0^4)$. At this stage, we mention that $p_0(=\sqrt{p_{0i}p_{0i}})$ is a magnitude of momentum p_{0i} but not a time component ϖ.

Finally, we find UV-propagators in the four dimensional frame with $p^\mu = (\varpi, 0, 0, p_3)$ as

$$t_{L/R} = -\frac{T_{L/R}}{\varpi^2 - c^2\Big(p_3^2 + \frac{2}{\omega}p_3^4 \mp \frac{8}{\eta^2\mu\omega}p_3^5 + \frac{128}{\eta^4\kappa^2\omega^2}p_3^6\Big)}. \tag{33}$$

3.2 GUP-corrected propagator

It is known for deformed HL gravity that the UV-propagator for tensor modes t_{ij} take a complicated form shown in Eq. (32), including up to p_0^6-term from the Cotton bilinear term $C_{ij}C_{ij}$.

At high energies, we assume that the UV-propagator takes the conventional form

$$G_{\rm UV}(\varpi, p^2) = \frac{1}{\varpi^2 - c^2p^2}, \tag{34}$$

whereas at low energies, the IR-propagator takes the conventional form

$$G_{\rm IR}(\varpi, p_0^2) = \frac{1}{\varpi^2 - c^2p_0^2}. \tag{35}$$

Considering (3), the UV-propagator (34) takes the form

$$G_{\rm UV}(\varpi, p_0^2) = \frac{1}{\varpi^2 - c^2\Big(p_0^2 + 2\beta p_0^4 + \beta^2 p_0^6\Big)}. \tag{36}$$

The GUP-corrected tensor propagator is determined by

$$t_{ij}^{GUP} = -G_{\rm UV}(\varpi, p_0^2)T_{ij} = -\frac{T_{ij}}{\varpi^2 - c^2\Big(p_0^2 + 2\beta p_0^4 + \beta^2 p_0^6\Big)}, \tag{37}$$

where scaling dimensions are given by $[\beta] = -2$, $[\varpi] = 3$, and $[c] = 2$ for the $z = 3$ HL gravity. *This is exactly the same form as the UV-tensor propagator (32) up to* p_0^4 when using the replacement of $\beta \to 1/\omega$ which was derived for entropy of the Kehagias-Sfetsos black hole without the Cotton tensor ($C_{ij} = 0$) [33]. However, considering terms beyond p_0^4 (p_0^5 and p_0^6), we could not make a definite connection between two propagators even though highest space derivative of sixth order are found in both propagators. Explicitly, the p_0^5-term is absent for the GUP-corrected propagator and coefficients in the front of p_0^6 are different. Two coefficients are the same for $\eta^4 = 128/\kappa^2$.

4 Discussions

We have explored a close connection between generalized uncertainty principle (GUP) and deformed Hořava-Lifshitz (HL) gravity. It was proposed that the GUP commutation relations describe the UV-quantum theory, while the canonical commutation relations represent the IR-quantum theory. Inspired by this UV/IR quantum mechanics, we obtain the GUP-corrected graviton propagator by introducing UV-momentum of $p_i = p_{0i}(1+\beta p_0^2)$ with p_{0i} the IR momentum. We compare this with tensor propagators in the HL gravity. Two are the same up to p_0^4-order, but the p_0^5-term arisen from the crossed term of Cotton and Ricci tensors did not appear in the GUP-corrected propagators.

Importantly, we confirm that the deformed HL gravity with ω parameter contains effects of quantum gravity implied by the GUP with the linear-order of β when using a relation of $\beta = 1/\omega$. This means that the deformed $z=2$ HL gravity without Cotten tensor could be well described by the GUP [2]. This Lagrangian is given by

$$\tilde{\mathcal{L}}_{z=2} = \sqrt{g}N\left[\frac{2}{\kappa^2}\Big(K_{ij}K_{ij} - \lambda K^2\Big) + \mu^4\Big(R + \frac{1}{2\omega}\frac{4\lambda-1}{3\lambda-1}R^2 - \frac{2}{\omega}R_{ij}R_{ij}\Big)\right]. \tag{38}$$

The tensor propagator is derived from the above Lagrangian on the Minkowski background where Ricci-square term R^2 does not contribute to the bilinear term of $t_{ij}t_{ij}$. Hence, it is easily shown that $\frac{2}{\omega}p_0^4$-term in the tensor propagator (32) comes from $R_{ij}R_{ij}$-term. On the other hand, the modified Heisenberg commutation relation (1) is satisfied to linear-order β when calculating the GUP-corrected propagator (37). Therefore, it is valid that the deformed $z=2$ HL gravity without Cotten tensor is well explained by the GUP.

However, it needs a further study in order to make a clear connection between $z=3$ HL gravity and the GUP with second-order of β (β^2) because the former contains the Cotton tensor C_{ij} and the replacement (3) is obscure.

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